

## Kinetics

By the end of this lesson, you should be able to:

- Calculate pos, vel, and acceleration caused by known forces
- At any point
- Of a Rigid body

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## Outline

- Equations
- Applications

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## Rotational Forces

$\Sigma F = ma$  may tell you about the motion of the COM,  
but how do you assess the rotation of points on the body?

$$\Sigma M = \frac{d}{dt}(I\omega) \quad \Sigma M = I\alpha \text{ If Inertia is constant}$$

$$I = \sum_i m_i r_i^2 = \int_m r^2 dm \quad I \text{ is the moment of inertia}$$

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## Rotational Forces

$$\Sigma M = I\alpha$$

$\Sigma M$  about the COM. This equation does not work for any other point, (without using parallel axis theorem)

If object is in equilibrium,  $\Sigma M$  about any point = 0

If object is not in equilibrium,  $\Sigma M = I\alpha$  only about COM, even if  $\alpha = 0$

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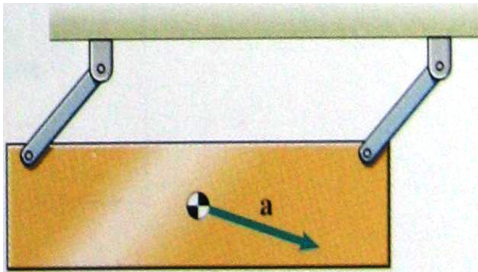
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## Translation

Only need  $F=ma$ ;  $\alpha = 0$ .

Can use  $\Sigma M=0$  to determine unknown forces

But remember – only about COM



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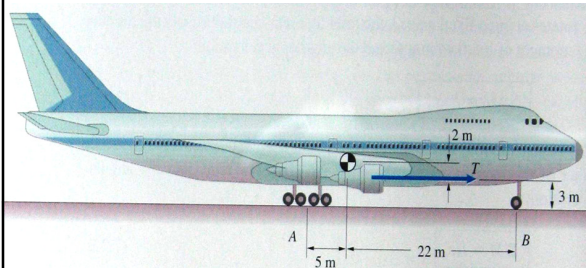
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## Example

The mass of the airplane is 250 Mg, and the thrust of its engines during takeoff is  $T = 700$  kN. Determine the airplane's acceleration, and the normal forces exerted on its wheels at A and B.



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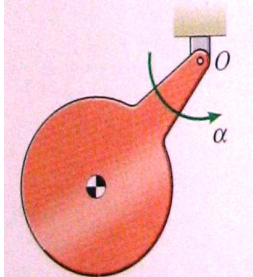
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## Rotation about a Fixed Axis

You only need  $\Sigma M = I\alpha$ , but you can use  $F = ma$  to determine unknown forces



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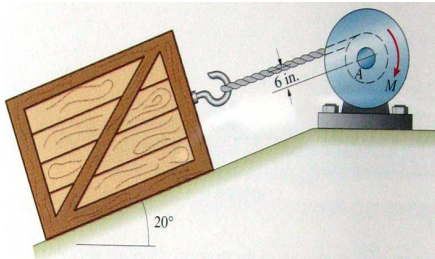
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## Example

The 100-kg crate is pulled up the inclined surface by a winch. The coefficient of kinetic friction between the crate and the surface is  $\mu_k = 0.4$ . The moment of inertia of the drum on which the cable is wound is  $I_A = 3 \text{ kg}\cdot\text{m}^2$ . If the motor exerts a couple  $M = 40 \text{ N}\cdot\text{m}$  on the drum, what is the crate's acceleration?



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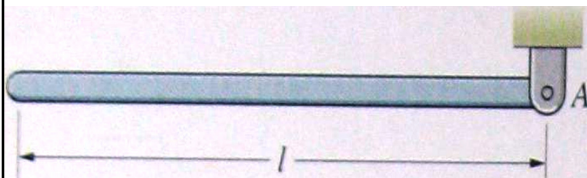
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## Example

The slender bar of mass  $m$  is released from rest in the horizontal position. At that instant, determine the bar's angular acceleration and the force exerted on the bar by the support A. The moment of inertia about the COM is:

$$I = \frac{1}{12} ml^2$$



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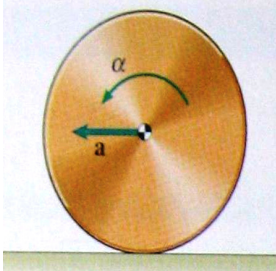
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## General Planar Motion

You need to use both  $\Sigma F=ma$  and  $\Sigma M=I\alpha$   
 You will also need equations from kinematics if COM motion is related to rotational motion



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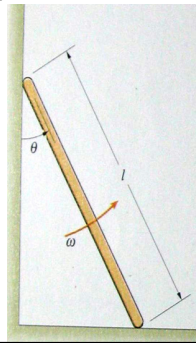
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## Example

The slender bar of mass  $m$  slides on the smooth floor and wall and has counterclockwise angular velocity  $\omega$  at the instant shown.

What is the bar's angular acceleration?




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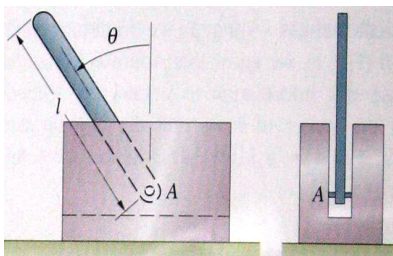
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## Example

The slender bar has mass  $m$  and is pinned at A to the metal block of mass  $m_B$  that rests on a smooth level surface. The system is released from rest in the position shown. What is the bar's angular acceleration at the instant of release?



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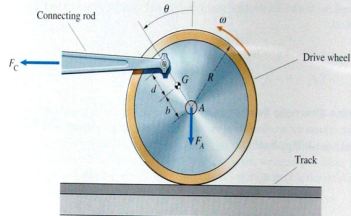
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## Example

The drive wheel rolls on the horizontal track. The wheel is subjected to a downward force  $F_A$  by its axle  $A$  and a horizontal force  $F_C$  by the connecting rod. The mass of the wheel is  $m$ , and the moment of inertia about its center is  $I$ . The center of mass  $G$  is offset a distance  $b$  from the wheel's center. At the instant shown, the wheel has a CCW angular velocity  $\omega$ . What is the wheel's angular acceleration?



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