Momentum Methods

By the end of this lesson, you should be able to:

- *Calculate the velocities resulting from a collision*
- *Determine the force exerted by mass-flow*

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Outline

- Energy vs. Momentum methods (review)
- Momentum method derivation
- Impulse
- Conservation of Linear Momentum
- Impacts
- Conservation of Angular Momentum
- Mass flow

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Newton's 2nd Law - Transformed

$$\underline{F} = \frac{d}{dt}(m\underline{v})$$

Energy

$$F \cdot v = \frac{d}{dt}(mv) \cdot v$$

$$\int \underline{F} \cdot d\underline{r} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Momentum

$$\int \underline{F}dt = \int d(m\underline{v})$$

$$\int \underline{F}dt = m\underline{v}_2 - m\underline{v}_1$$

Only for constant mass

Both relate integral of external forces to change in object's velocity

Energy	Momentum
Collapses to Scalar	Preserves Vector
Useful for:	Useful for:
•When external forces are <u>known</u> function of <u>position</u>	•When external forces are <u>known</u> function of <u>time</u>
• Systems with no net work	• Collisions
	•Changes in mass

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Momentum Method - Derivation

$$\underline{F} = \frac{d}{dt}(m\underline{v})$$

$$\underline{F} = m\frac{d\underline{v}}{dt}$$ Only for constant mass

$$\int_{t_1}^{t_2} \underline{F}dt = m\underline{v}_2 - m\underline{v}_1$$

Impulse — Momentum

$$(t_2 - t_1)\sum \underline{F}_{av} = m\underline{v}_2 - m\underline{v}_1$$

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Conservation of Linear Momentum

If external forces can be neglected:

Total linear momentum of the objects is conserved:

$$m_A\underline{v}_A + m_B\underline{v}_B = \text{constant}$$

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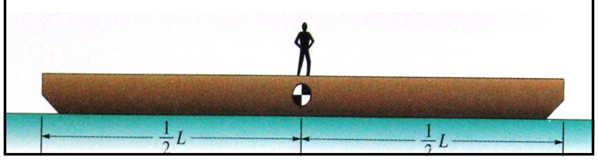
Example

A person of mass m_P stands at the center of a stationary boat of mass m_B. Neglect horizontal forces exerted on the boat by the water.

a) If the person starts running to the right with velocity v_P relative to the water, what is the resulting velocity of the barge relative to the water?

b) If the person stops when he reaches the right-hand end of the barge, what are his position and the barge's position relative to their original positions?



Impacts

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

Coefficient of restitution: $e = \dfrac{v_B' - v_A'}{v_B - v_A}$

If e=0, impact is perfectly plastic: objects stick together

If e = 1, impact is perfectly elastic: total kinetic energy is the same before and after the impact

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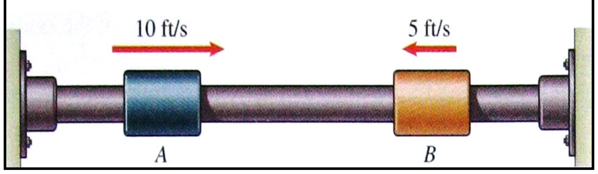
Example

The two 10-lb weights slide on the smooth horizontal bar. Determine their velocities after they collide:

a) if they are coated with Velcro and stick together;

b) if the coefficient of restitution is e=.8



Angular Momentum

$$\underline{H} = \underline{r} \times m\underline{v}$$

$$\int_{t_1}^{t_2} \left(\underline{r} \times \Sigma \underline{F} \right) dt = \underline{H}_2 - \underline{H}_1$$

Angular Impulse — Angular momentum

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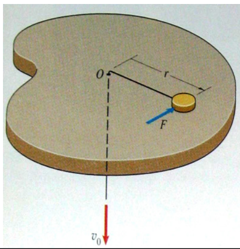
Example

A disk of mass m attached to a string slides on a smooth horizontal table under the action of a constant transverse force F. The string is drown through a hole in the table at O at constant velocity v_0. At t = 0, $r=r_0$ and the transverse velocity of the disk is zero. What is the disk's velocity as a function of time?



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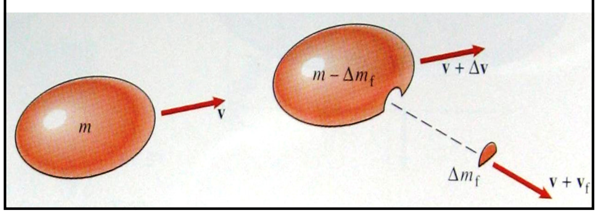
Mass Flow

$$F_f = -\frac{dm_f}{dt} v_f$$



Example

The rocket sled is being slowed by a water brake after its rocket motor has burned out. A tube extends from the sled into a trough of water with its open end pointing forward, so that water enters the tube in the direction parallel to the x axis as the sled moves forward. The other open end of the tube points upward, so that the water flows out in the direction parallel to the y axis. If the sled's velocity is v, the water enters the tube with velocity v relative to the sled and flows out with the same velocity. The mass flow rate of water through the tube is ρvA, where ρ=1.94 slug/ft3 is the mass density of the water and A = 0.1 ft² is the cross-sectional area of the tube. At the instant when v = 1000 ft/s, what forces are exerted on the sled by the flows of water entering and leaving it?