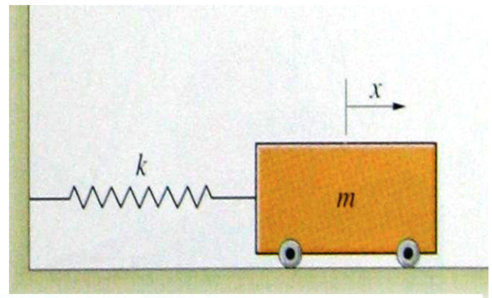
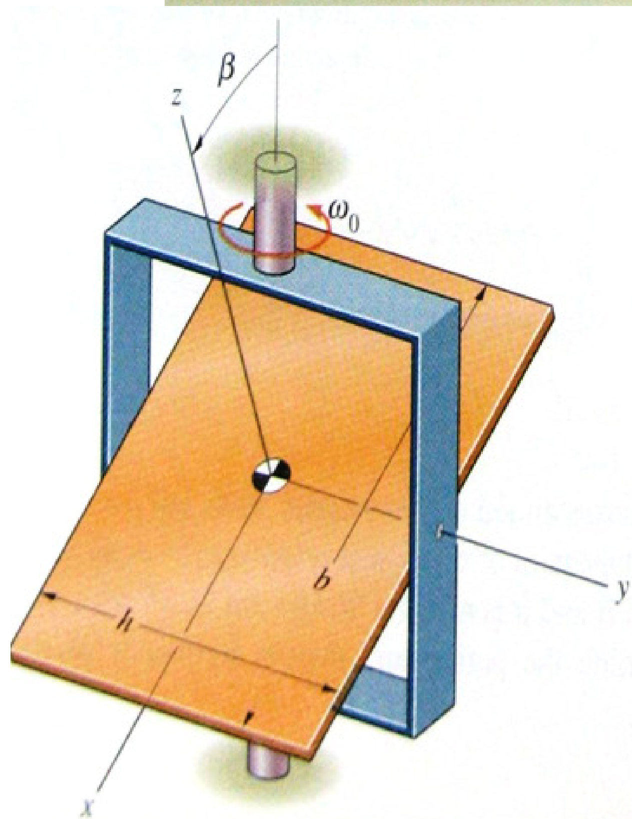
EGBE261: Biomechanics 2
Homework from Chapter 10: Vibrations

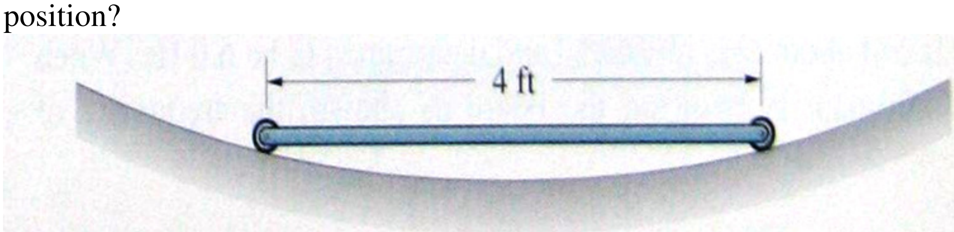
10.6 The mass m = 10 kg and k = 90 N/m. The coordinate x measures the displacement of the mass relative to its equilibrium position. At t = 0, the mass is released from rest in the position x = 0.1 m. a) Determine the period and natural frequency of the resulting vibrations. B) Determine x as a function of time.



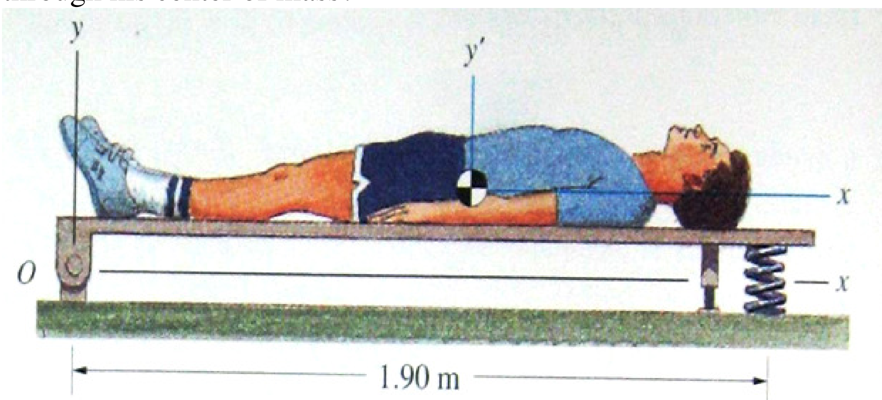
10.10. The thin rectangular plate is attached to the rectangular frame by pins. The frame rotates with constant angular velocity ω0 = 6 rad/s. The angle β between the z axis of the body-fixed coordinate system and the vertical is governed by the equation $\frac{d^2\beta}{dt^2} = -\omega_0^2 \sin\beta\cos\beta$. At t = 0, the angle β = 0.01 rad and dβ/dt = 0. Determine β as a function of time.

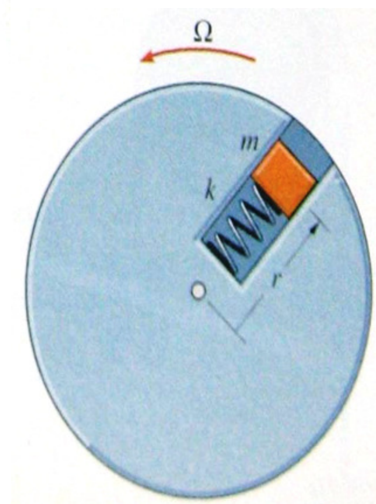


10.20. The slender bar has roller supports at its ends and is at rest in a circular depression with an 8-ft radius. What is the frequency of small vibrations of the bar relative to its equilibrium position?

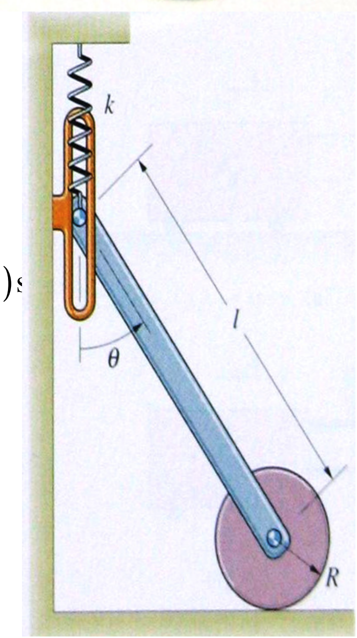


10.24. Engineers use the device shown to measure an astronaut's moment of inertia. The horizontal board is pinned at O and supported by the linear spring with constant k = 12 kN/m. When the astronaut is not present, the frequency of small vibrations of the beard about O is measured and determined to be 6.0 Hz. When the astronaut is lying on the board as shown, the frequency of the small vibrations of the board about O is 2.8 Hz. The astronaut's center of mass is at x = 1.01 m, y = 0.16 m, and his mass is 81.6 kg. What is his moment about the z' axis through his center of mass?





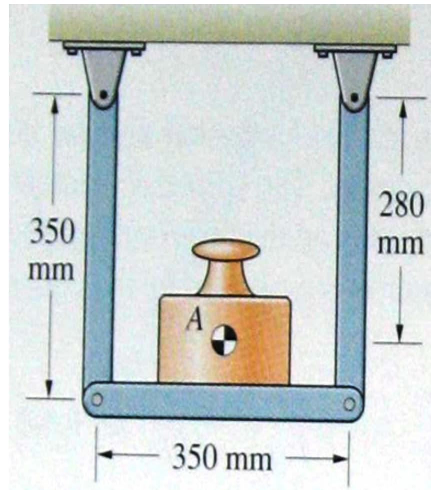
10.26. A disk rotates about a fixed vertical axis with constant angular velocity Ω. A mass m slides in a smooth slot in the disk and is attached to a spring with constant k. The distance from the center of the disk to the mass when the spring is unstretched is r0. Show that if k/m > Ω2, the natural frequency of vibration of the mass is $f = (1/2\pi)\sqrt{k/m - \Omega^2}$.



10.32 The masses of the slender bar and the homogeneous disk are m and md, respectively. The spring is unstretched when θ=0. Assume that the disk rolls on the horizontal surface. Show that the motion is governed by the equation

$$\left(\frac{1}{3}+\frac{3m_d}{2m}\cos^2\theta\right)\frac{d^2\theta}{dt^2}-\frac{3m_d}{2m}\sin\theta\cos\theta\left(\frac{d\theta}{dt}\right)^2-\frac{g}{2l}\sin\theta+\frac{k}{m}(1-\cos\theta)$$

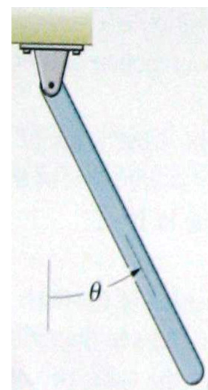
10.34. The mass of each slender bar is 1 kg. If the natural frequency of small vibrations of the system is 0.935 Hz, what is the mass of the object A?



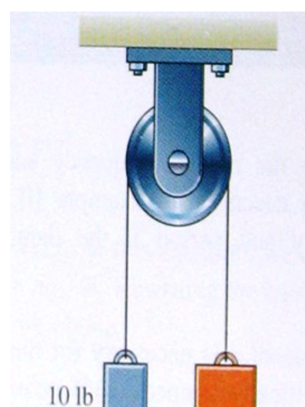
10.44. The homogeneous slender bar is 4 ft long and weighs 10 lb. Aerodynamic drag and friction at the support exert a resisting moment on the bar of magnitude 0.5(dθ/dt) ft-lb, where dθ/dt is the angular velocity of the bar in rad/s.

a) What are the period and natural frequency of small vibrations of the bar?

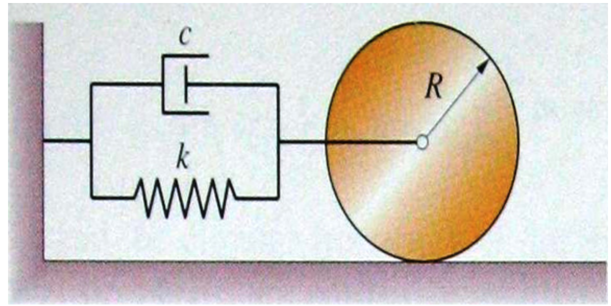
b) How long does it take for the amplitude of vibration to decrease to one-half of its initial value?



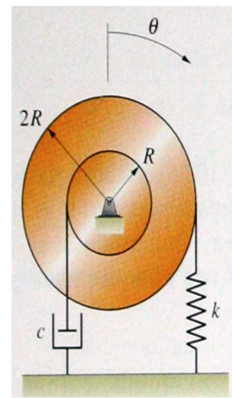
10.50. The spring constant is k = 30 lb/ft and the damping constant is c = 3.5 lb-s/ft. The radius of the pulley is 0.5 ft, and its moment of inertia is 0.25 slug-ft^2. The system is released from rest with the spring unstretched. Determine the position of the 10-lb weight relative to its position at t=0 as a function of time.



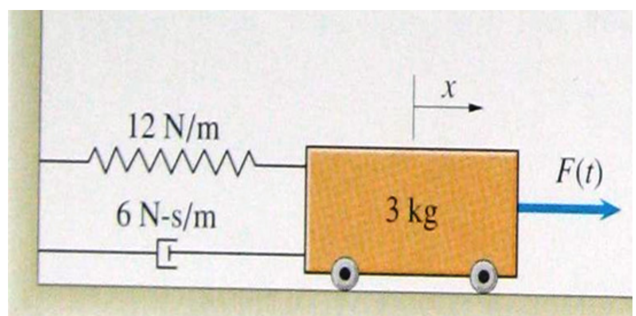
10.52. The homogeneous disk weighs 100 lb and its radius is R = 1 ft. It rolls on the plane surface. The spring constant is k = 100 lb/ft and the damping constant is c = 3 lb-s/ft. The spring is unstretched at t = 0 and the disk has a clockwise angular velocity of 2 rad/s. What is the angular velocity of the disk when t = 3 s?



10.54. Let θ be the angular displacement of the disk relative to its position when the spring is unstretched. The radius R = 250 mm, k = 150 N/m, and the moment of inertia is I = 2 kg-m^2.
a) What value of c will cause the system to be critically damped?
b) at t = 0, the spring is unstretched and the clockwise angular velocity of the disk is 10 rad/s. Determine θ as a function of time.
c) Using the result of b), determine the maximum resulting angular displacement of the disk and the time at which it occurs.



10.60. The damped spring-mass oscillator is initially stationary with the spring unstretched. At t = 0, a constant force F(t) = 6N is applied to the mass. A) What is the steady-state (particular) solution? B) Determine the position of the mass as a function of time.



10.66. A 1.5-kg cylinder is mounted on a "sting" in a wind tunnel with the cylinder axis transverse to the flow direction. When there is no flow, a 10-N vertical force applied to the cylinder causes it to deflect 0.15 mm. When air flows in the wind tunnel, vortices subject the cylinder to alternating lateral forces. The velocity of the air is 5 m/s, the distance between vortices is 80 mm, and the magnitude of the lateral forces is 1 N. If you model the lateral forces by the oscillatory function F(t) = $\sin\omega_0 t$ N, what is the amplitude of the steady-state lateral motion of the sphere?