Virtual Work

By the end of this lesson, you should be able to:

- *<u>Efficiently</u> calculate desired variables in machines*

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Outline

- Work
- Virtual Work

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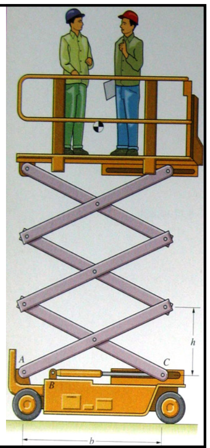
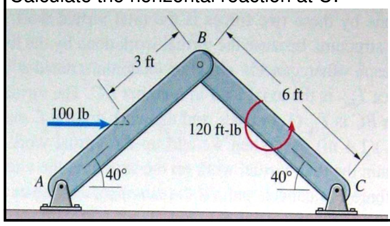
Virtual Work

Calculate the axial force of the hydraulic cylinder:



Calculate the horizontal reaction at C:



Virtual Work

- Good for machines with simple geometry
- Eliminates undesired reactions and internal forces
- You only have to deal with:
  - Loads
  - Applied Forces
  - Friction Forces

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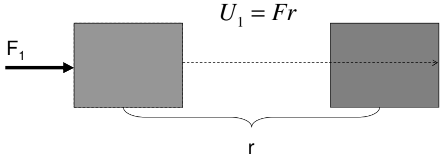
Work

$$U = \underline{F} \cdot \underline{r}$$
$$U = |\underline{F}||\underline{r}|\cos\theta$$

$$U_1 = Fr$$



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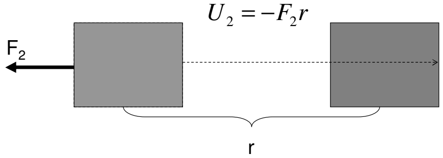
Work

$$U = \underline{F} \cdot \underline{r}$$
$$U = |\underline{F}||\underline{r}|\cos\theta$$

$$U_2 = -F_2 r$$



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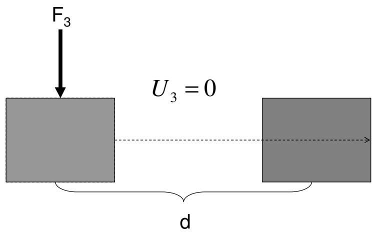
Work

$$U = \underline{F} \cdot \underline{r}$$
$$U = |\underline{F}||\underline{r}|\cos\theta$$



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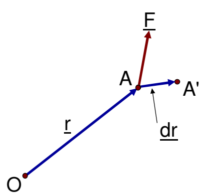
Little Work

$$U = \underline{F} \cdot \underline{r}$$
$$dU = \underline{F} \cdot \underline{dr}$$



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Little Work (angular)

$$U = \underline{M} \cdot \theta$$
$$dU = \underline{F} \cdot \underline{d\theta}$$

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Virtual Work

If an object is in equilibrium, the total virtual work of the external forces acting on the object are 0 for any virtual displacement of the particle

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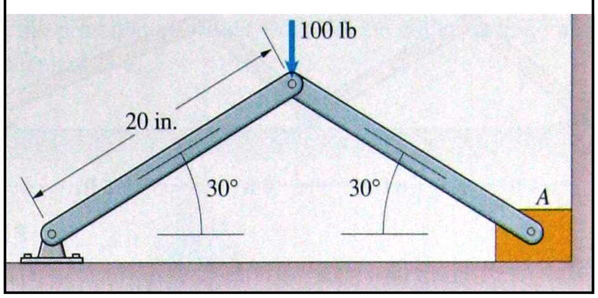
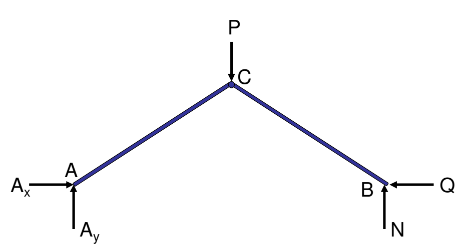
Example (p. 562)

What is the force exerted on the wall?



Solution



5 unknowns – not a nice problem

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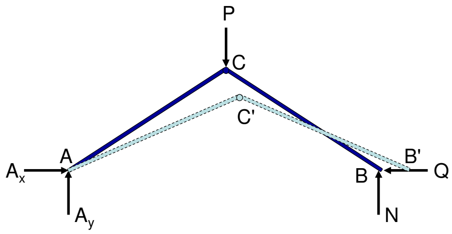
Solution: Virtual work



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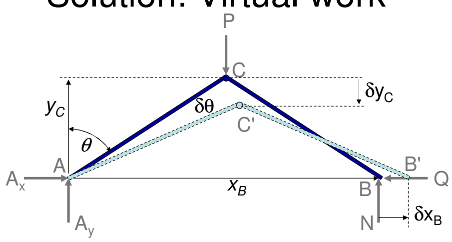
Solution: Virtual work



$$\delta U_P = +P(-\delta y_C) = -P\delta y_C$$
$$y_C = l\cos\theta$$
$$\delta y_C = -l\sin\theta$$

$$\delta U_Q = -Q(\delta x_B) = -Q\delta x_B$$
$$x_B = 2l\sin\theta$$
$$\delta x_B = 2l\cos\theta$$

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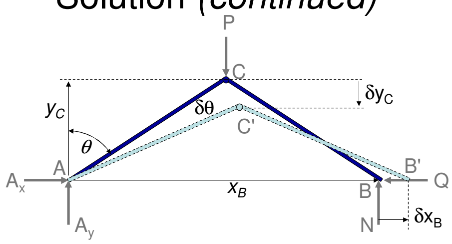
Solution *(continued)*



$$\delta U = \delta U_P + \delta U_P = -P\delta y_C - Q\delta x_B$$
$$\delta U = Pl\sin\theta\delta\theta - 2Ql\cos\theta\delta\theta$$
$$2Ql\cos\theta\delta\theta = Pl\sin\theta\delta\theta$$
$$Q = \frac{1}{2}P\tan\theta$$

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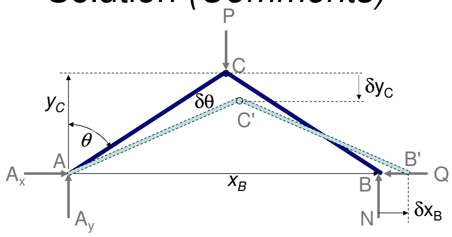
Solution *(Comments)*



Solution was easy because geometry was easy

$y_C = l\cos\theta \qquad x_B = 2l\sin\theta$

$\delta y_C = -l\sin\theta \qquad \delta x_B = 2l\cos\theta$

Solution was much easier than standard equilibrium equations

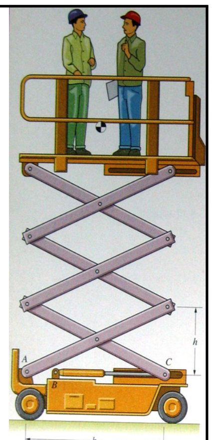
a

Example

The total weight of the platform and men is W.

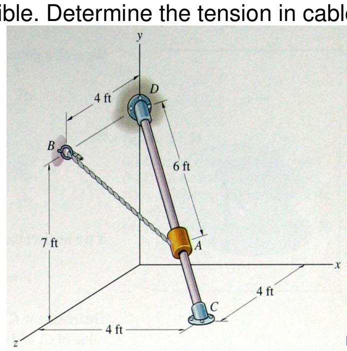
Ignore the weight of the beams.

What force must the hydraulic cylinder exist to maintain equilibrium?

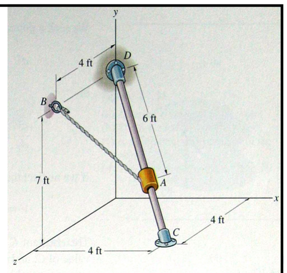


Example

The collar A weighs 100 lb, and friction is negligible. Determine the tension in cable AB



Strategy



Let *s* be the distance from D to the collar

Calculate e_{DC}

Let the collar undergo a virtual displacement: $\delta s\, e_{DC}$

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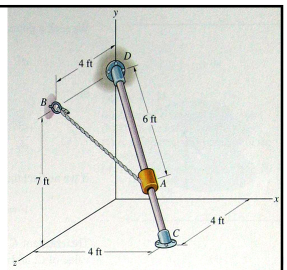
Solution



$$\underline{DC} = 4\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|\underline{DC}| = 9$$

$$\hat{e}_{DC} = \frac{4}{9}\hat{i} - \frac{7}{9}\hat{j} + \frac{4}{9}\hat{k}$$

$$\underline{DA} = 6\hat{e}_{DC} = \frac{8}{3}\hat{i} - \frac{14}{3}\hat{j} + \frac{8}{3}\hat{k}$$

$$A = OD + DA = \frac{8}{3}\hat{i} + \frac{7}{3}\hat{j} + \frac{8}{3}\hat{k}$$

$$\underline{AB} = -\frac{8}{3}\hat{i} + \frac{14}{3}\hat{j} + \frac{4}{3}\hat{k}$$

$$|\underline{AB}| = 5.53$$

$$\hat{e}_{AB} = -.48\hat{i} + .82\hat{j} + 24\hat{k}$$

$$\underline{T} = T\hat{e}_{AB}$$

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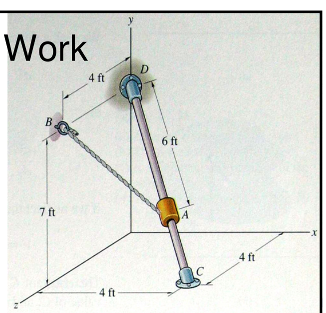
Solution: Virtual Work



$$\hat{e}_{DC} = \frac{4}{9}\hat{i} - \frac{7}{9}\hat{j} + \frac{4}{9}\hat{k}$$

$$\hat{e}_{AB} = -0.48\hat{i} + .84\hat{j} + 0.24\hat{k}$$

$$\underline{W} = -100\hat{j}$$

$$\delta U = W \cdot \delta s \hat{e}_{DC} + (T\hat{e}_{AB}) \cdot \delta s \hat{e}_{DC} = 0$$

$$T = -\frac{W \cdot \delta s \hat{e}_{DC}}{\hat{e}_{AB} \cdot \delta s \hat{e}_{DC}} = -\frac{W \cdot \hat{e}_{DC}}{\hat{e}_{AB} \cdot \hat{e}_{DC}}$$

$$W \cdot \hat{e}_{DC} = (-100)\left(-\frac{7}{9}\right) = 77.78$$

$$\hat{e}_{AB} \cdot \hat{e}_{DC} = (-0.48)\frac{4}{9} - (0.84)\frac{7}{9} + (0.24)\frac{4}{9} = -.76$$

$$T = -\frac{W \cdot \hat{e}_{DC}}{\hat{e}_{AB} \cdot \hat{e}_{DC}} = -\frac{77.78}{-.76} = 102 \quad lb$$

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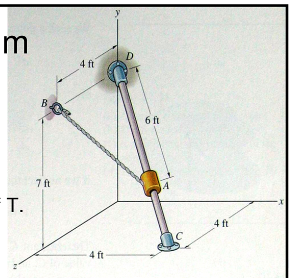
Solution: Equilibrium



$\hat{e}_{DC} = \frac{4}{9}\hat{i} - \frac{7}{9}\hat{j} + \frac{4}{9}\hat{k}$

$\hat{e}_{AB} = -0.48\hat{i} + .84\hat{j} + 0.24\hat{k}$

$\underline{W} = -100\hat{j}$

1) Find Direction e_q of Reaction (Cross product). e_q is a function of T.

$\underline{q} = (\underline{W} + \underline{T}) \times \hat{e}_{DC}$

$\hat{e}_q = \frac{\underline{q}}{|\underline{q}|}$

2) Scale q by R to obtain Reaction Force

$\underline{R} = R\hat{e}_q$

3) Solve Force Equilibrium Equations *($\underline{T}$+$\underline{W}$+$\underline{R}$=0)*.

Moment equations are probably worthless, because system is concurrent at A: moment = 0

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Typical Exam question

- None.
  I will never intentionally ask you a Virtual Work question.
- But, it is useful for solving normal questions of Equilibrium.

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