Moments & Torque

By the end of this lesson, you should be able to:

- *Calculate the moment of any force about any point*
- *Simplify systems of forces & torques*

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Outline

- Properties of Moment / Torque
- Calculation of Torque in 2D
- Calculation of Torque in 3D using vectors
- Couples (Moments with no net force)
- Simplifying complex systems

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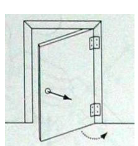
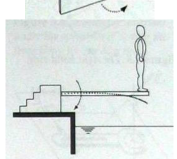
Torque vs. Moment

- Torque: The rotational and twisting actions of applied forces
- Moment: The bending effect of applied forces
- *Same equation*





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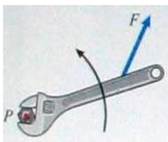
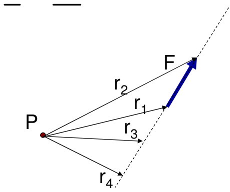
Moment

NOT $\underline{F} \times \underline{r}$!

$$M_p = \underline{r} \times \underline{F}$$





<u>If you use the Cross product</u> (not trig), any <u>r</u> that goes from point P to the line of action of force F is acceptable.
r does not have to be perpendicular to F.

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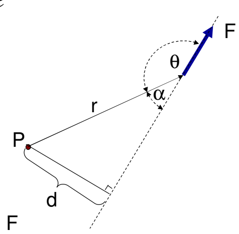
Moment: Cross Product ➔ trig

$$\begin{aligned}\underline{M}_p &= \underline{r} \times \underline{F} \\ &= |\underline{r}||\underline{F}|\sin\theta\hat{e} \\ &= |\underline{r}||\underline{F}|\sin(180-\alpha)\hat{e} \\ &= |\underline{r}||\underline{F}|\sin(\alpha)\hat{e} \\ &= |\underline{r}|\sin(\alpha)|\underline{F}|\hat{e} \\ &= d|\underline{F}|\hat{e}\end{aligned}$$



$M_p = dF$, where d is the perpendicular distance from P to F

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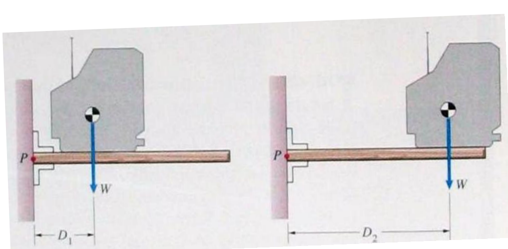
Magnitude



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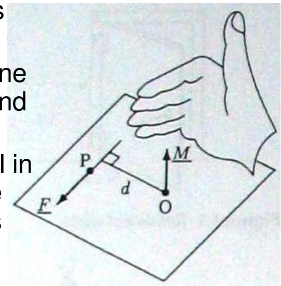
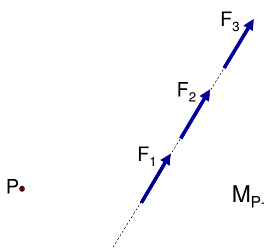
Direction

- Counter-clockwise is positive
- Perpendicular to plane on which the point and force lie
- Right hand rule: Curl in the direction that the applied force rotates the body about O



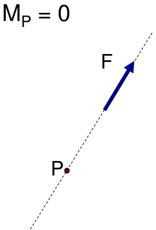
Properties: Invariant to position on line of action



$M_{P\text{-}1} = M_{P\text{-}2} = M_{P\text{-}3}$

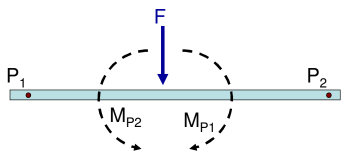
Properties: About a point

$d = 0 \rightarrow M_P = 0$



Location

A force applied to a body may rotate the body in 1 direction with respect to 1 point and in the opposite direction with respect to another point



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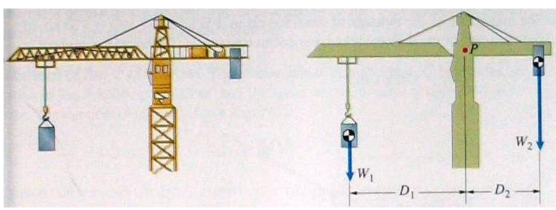
Net Moment



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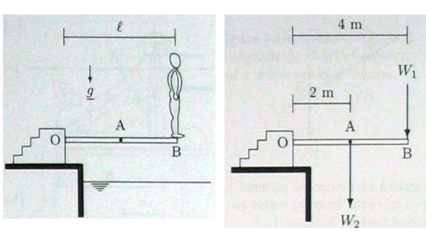
Example

Diving board has a mass of 120 kg and is 4m long. The person has a mass of 90 kg. Determine the moments generated at point O.



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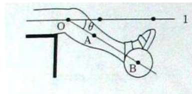
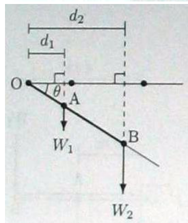
Example

Consider an athlete wearing a weight boot. The weight of the athlete's lower leg is W_1=50N and the weight of the boot is W_2 = 100 N. As measured from the knee joint at O, the center of mass (A) of the lower leg is located at a distance a=20 cm and the center of mass (B) of the weight boot is located at a distance b=50 cm.





Determine the net moment generated about the knee joint when the lower leg is extended horizontally (position 1) and when the lower leg makes an angle of 30°

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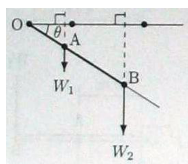
Solution, θ=0°



$$\sum M = -d_1 W_1 - d_2 W_2$$

$$\sum M = -0.2 * 50 - 0.5 * 100 (Nm)$$

$$\sum M = -60 Nm \quad ccw$$

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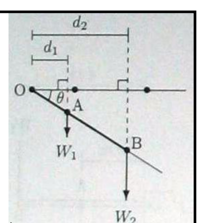
Solution, θ=30°



$$\sum M = -d_1 W_1 - d_2 W_2$$

$$\sum M = -0.2\cos(30) * 50 - 0.5\cos(30) * 100 (Nm)$$

$$\sum M = -52 Nm \quad ccw$$

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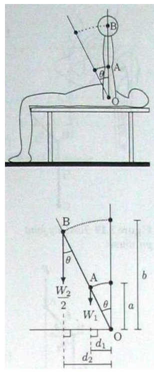
Example

O represents the shoulder joint. A is the center of gravity of one arm, and B is a point of intersection of the centerline of the barbell and the extension of line OA.
The distance between O and A is a=24 cm and the distance between O and B is b=60 cm. Each arm weighs W1=50 N and the total weight of the barbell is W2=300 N.

Determine the net moment about the shoulder joint for θ=0° and 30°.



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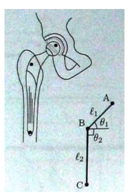
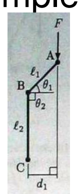
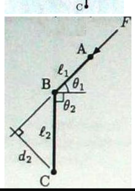
Hip prosthesis Example

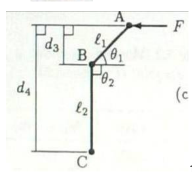
l_1=50mm, l_2=100 mm, θ_1=45°, and θ_2=90°. Assume that when standing symmetrically on both feet, a joint reaction force of F=400 N is acting at the femoral head due to the body weight of the patient.

Determine the moments generated about points B on the prostheses for 3 different lines of action.









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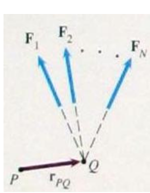
Varignon's Theorem

- Let F_1, F_2, … F_N be a concurrent system of forces whose lines of action intersect at point Q. The moment about a point P is:

$$r_{PQ} \times (F_1 + F_2 + \dots F_N)$$



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Using Vectors

1) Choose the vector r = you must choose a position vector from P to any point on the line of action F and determine its components
2) Evaluate r x F

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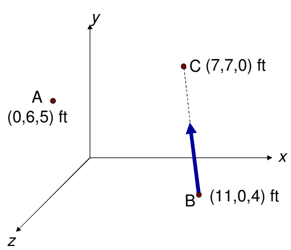
Example

The line of action of the 90 N force passes through points B and C. What is the moment of F about point A?



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Solution

1) Determine Vector F

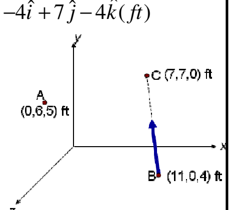
a) Find BC

$$\underline{BC} = (7-11)\hat{i} + (7-0)\hat{j} + (0-4)\hat{k} = -4\hat{i} + 7\hat{j} - 4\hat{k}\,(ft)$$

b) Find $\hat{e}_{BC}$

$$|\underline{BC}| = \sqrt{(-4)^2 + 7^2 + (-4)^2} = 9$$

$$\hat{e}_{BC} = -\frac{4}{9}\hat{i} + \frac{7}{9}\hat{j} - \frac{4}{9}\hat{k}$$



c) Scale $\hat{e}_{BC}$ by force magnitude to find F

$$\underline{F} = 90\hat{e}_{BC} = -40\hat{i} + 70\hat{j} - 40\hat{k}\,(lb)$$

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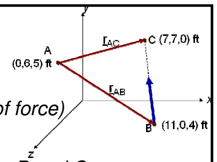
Solution using $\underline{r}_{AB}$



2) Determine Vector $\underline{r}$
(point A to any point on line of action of force)

2 known points on the line of action are B and C. To demonstrate that you can choose any point, we will use both $\underline{r}$=AB and $\underline{r}$=BC

$$\underline{r}_{AB} = (11-0)\hat{i} + (0-6)\hat{j} + (4-5)\hat{k} = 11\hat{i} - 6\hat{j} - \hat{k}(ft)$$

3) Evaluate $\underline{M} = \underline{r} \times \underline{F}$

$$M_A = \underline{r}_{AB} \times \underline{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{AB\,x} & r_{AB\,y} & r_{AB\,z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -6 & -1 \\ -40 & 70 & -40 \end{vmatrix}$$

$$= 310\hat{i} + 480\hat{j} + 530\hat{k}(ft-lb)$$

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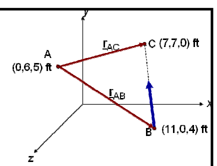
Solution using $\underline{r}_{AC}$



2) Determine Vector $\underline{r}$

$$\underline{r}_{AC} = (7-0)\hat{i} + (7-6)\hat{j} + (0-5)\hat{k} = 7\hat{i} + \hat{j} - 5\hat{k}(ft)$$

3) Evaluate $\underline{M} = \underline{r} \times \underline{F}$

$$M_A = \underline{r}_{AC} \times \underline{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{AC\,x} & r_{AC\,y} & r_{AC\,z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -1 & -5 \\ -40 & 70 & -40 \end{vmatrix}$$

$$= 310\hat{i} + 480\hat{j} + 530\hat{k}(ft-lb)$$ *Same answer*

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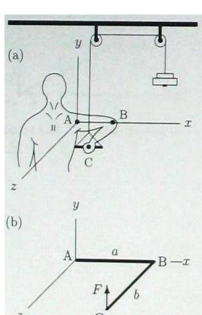
Example

The weight applies an upward force with magnitude F on the arm at point C. The lengths of the upper arm and lower arm are a=25 cm and b=30 cm, and the magnitude of the applied force = 200 N. Determine the magnitudes and directions of moments developed at the forearm & arm by F.



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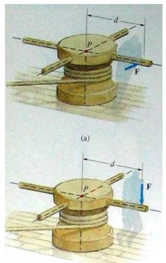
Moment of a Force about an axis

Sometimes you only care about the component of a torque in a certain direction



$\underline{r} \times \underline{F}$ The entire moment

$\left[\hat{e} \cdot \left(\underline{r} \times \underline{F}\right)\right]\hat{e}$ The component of the moment in the direction of an axis

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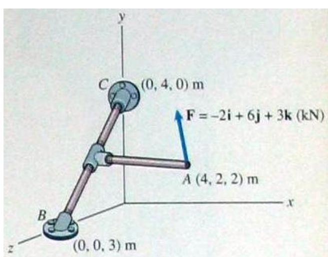
Example

- What is the moment of F about bar BC?



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Solution

1) Determine Vector r
(any point on BC to any point on line of action of force)

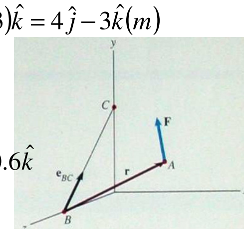
$$r = BA = (4-0)\hat{i} + (2-0)\hat{j} + (2-3)\hat{k} = 4\hat{i} + 2\hat{j} - \hat{k}\,(m)$$

2) Determine Vector ê along BC

$$\underline{BC} = (0-0)\hat{i} + (4-0)\hat{j} + (0-3)\hat{k} = 4\hat{j} - 3\hat{k}\,(m)$$

$$\left|\underline{BC}\right| = \sqrt{0^2 + 4^2 + (-3)^2} = 5$$

$$\hat{e}_{BC} = \frac{\underline{BC}}{\left|\underline{BC}\right|} = \frac{4\hat{j} - 3\hat{k}}{5} = 0.8\hat{j} - 0.6\hat{k}$$



Solution *(continued)*

3) Evaluate M_L using mixed triple product

$$\hat{e}_{BC}\cdot(\underline{r}\times\underline{F})=\begin{vmatrix}\hat{e}_{BC\,x} & \hat{e}_{BC\,y} & \hat{e}_{BC\,z}\\ r_x & r_y & r_z\\ F_x & F_y & F_z\end{vmatrix}=\begin{vmatrix}0 & .8 & .6\\ 4 & 2 & -1\\ -2 & 6 & 3\end{vmatrix}=-24.8kNm$$

$$\underline{M}_{BC}=[\hat{e}_{BC}\cdot(\underline{r}\times\underline{F})]\hat{e}_{BC}=-24.8\hat{e}_{BC}(kNm)=-19.84\hat{j}+14.88\hat{k}$$

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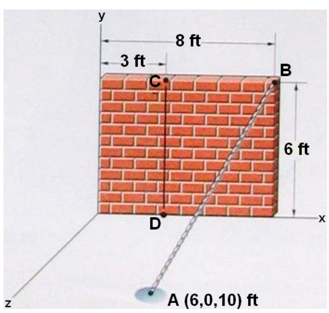
Example

The tension in AB = 80 lb

What is the moment about line CD?



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