Applications of Plane Stress

By the end of this lesson, you should be able to:

- *Apply the concepts you learned last week to more interesting and realistic problems*

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Outline

- Pressure Vessels
  - Spherical Pressure Vessels
  - Cylindrical Pressure Vessels
- Combined Loadings

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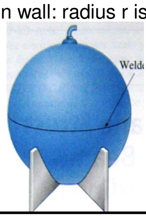
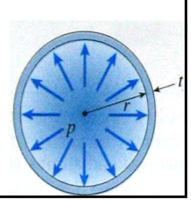
Pressure Vessels

- Examples:
  - Container of pressurized oxygen
  - Container of fluid for blood or renal dialysis
  - Red Blood cells

We will only look at thin-walled pressure vessels
Thin wall: radius r is >10 * thickness t





Spherical Pressure Vessels

Pressure force P = p(πr^2),
where p is the net internal pressure
(gauge pressure = internal – external pressure)

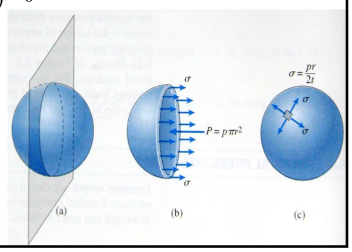
Shear force = $\sigma(2\pi r_m t)$, where $r_m = r + t/2$

$$\sum Fx = \sigma\left(2\pi r_m t\right) - p\left(\pi r^2\right) = 0$$

$$\sigma = \frac{pr^2}{2r_m t}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \qquad \sigma_3 = 0$$

$$\tau_{max} = \frac{\sigma}{2} = \frac{pr}{4t}$$ At 45°

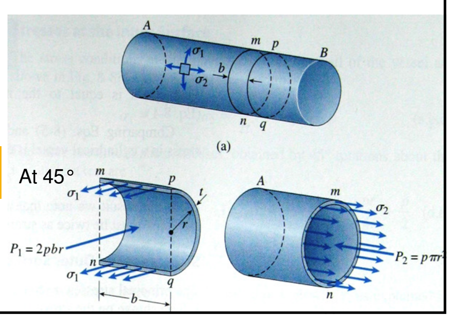


Cylindrical Pressure Vessels

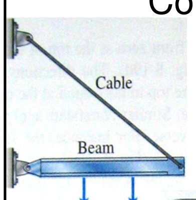
$$\sigma_1 = \frac{pr}{t}$$

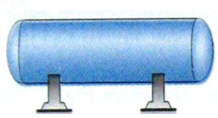
$$\sigma_2 = \frac{pr}{2t}$$

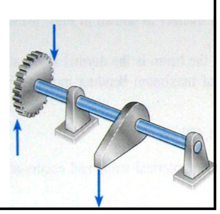
$$\tau_{max} = \frac{pr}{2t}$$ At 45°



Combined Loading







Steps

1) Select a *good* point (difficult!)
2) For each load, determine stress resultants:
   - Axial force
   - Twisting moment
   - Bending moment
   - Shear force
   - Internal pressure
3) Calculate normal or shear stresses for each stress
4) Combine individual stresses to find σ_x,σ_y, and τ_{xy}
5) Determine principle stresses
6) Determine strains

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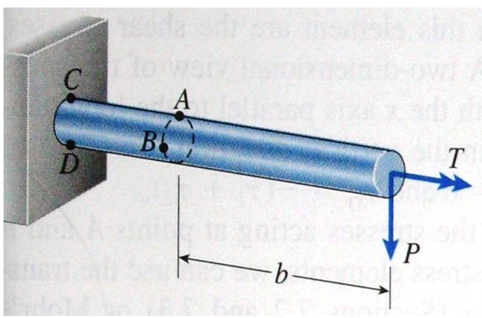
Example



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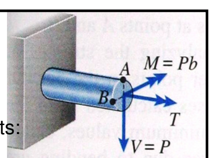
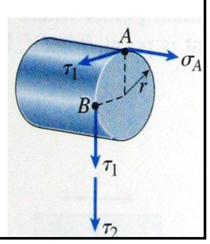
Solution



1) Points were picked for us:
2) For each load, determine stress resultants:
   - Axial force: *none*
   - Twisting moment: torque T: τ_1 at A and B
   - Bending moment: Load P: σ_A at A
   - Shear force: Load P: τ_2 at B
   - Internal pressure: *none*



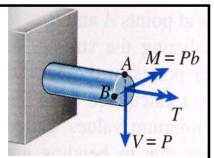
Solution (cont)

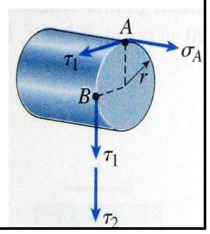
3) Calculate stresses

$$\tau_1 = \frac{Tr}{I_p} = \frac{2T}{\pi r^3}$$

$$\sigma_A = \frac{Mr}{I} = \frac{4M}{\pi r^3}$$

$$\tau_2 = \frac{4V}{3A} = \frac{4V}{3\pi r^2}$$



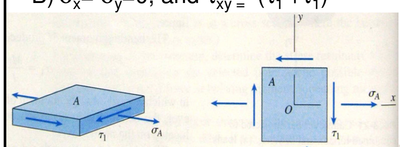


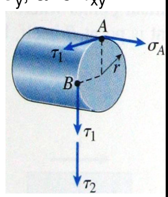
Solution (cont)

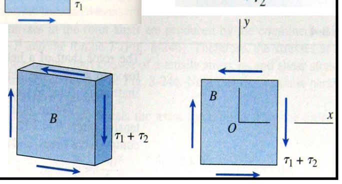
4) Combine individual stresses to find σ_x,σ_y, and τ_{xy}

A) $\sigma_x = \sigma_A$, $\sigma_y=0$, and $\tau_{xy} = -\tau_1$

B) $\sigma_x = \sigma_y=0$, and $\tau_{xy} = -(\tau_1 + \tau_1)$







Solution (continued)

- Use Transformation equations or Mohr's circle to determine principal stresses, maximum shear stresses, and stresses acting in inclined direction
- Use Hook's law to determine strains at A and B

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Critical Points

- Usually, we want to pick the location, and points, that give us maximum stress.
- We can use:
  - Good judgment ☺
  - Large number of points
  - Equation derivation
  - Simplifying assumptions

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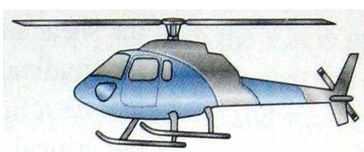
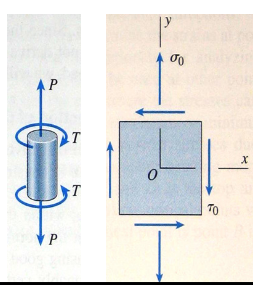
Example

- The rotor shaft drives the rotor blades that provide the lifting force to support the helicopter in the air. The shaft is accordingly subjected to a combination of torsion and axial loading
- For a 50-mm diameter shaft transmitting a torque T = 2.4 kNm and a tensile force P = 125 kN, determine:
  - Maximum tensile stress
  - Maximum compressive stress
  - Maximum shear stress



Solution

1) Select a *good* point: any point on surface
2) For each load, determine stress resultants:
  - Axial force: P: σ_0
  - Twisting moment: T: τ_0
  - Bending moment: none
  - Shear force: none
  - Internal pressure: none



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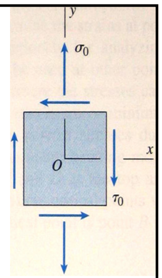
Solution (continued)

3) Calculate stresses

$$\sigma_0 = \frac{P}{A} = \frac{125kN}{\pi(0.025m)^2} = 63.66MPa$$

$$\tau_0 = \frac{Tr}{I_p} = \frac{(2.4kNm)}{\pi(0.025m)^3} = 97.78MPa$$



4) Combine individual stresses to find σ_x,σ_y, and τ_{xy}

σ_x= 0

σ_y= σ_0= 63.66 MPa

τ_{xy} = -τ_0 = -97.78 MPa

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Solution (continued)

- Maximum Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 + 63.66MPa}{2} \pm \sqrt{\left(\frac{0 - 63.66MPa}{2}\right)^2 + (-97.78MPa)^2}$$

$$= 32MPa \pm 103MPa$$

σ_1 = 135 MPa *σ_2=-71 MPa*

- Maximum In-plane Shear Stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 103MPa$$

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Solution (continued)

σ1 and σ2 have different signs, so maximum shear stress will be in-plane.

(Don't forget to at least think about out-of-plane shear stress!)

$$\tau_{(max)_{x_1}} = \pm\frac{\sigma_2}{2}$$

$$\tau_{(max)_{y_1}} = \pm\frac{\sigma_1}{2}$$

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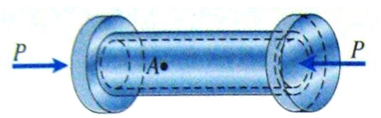
Example

- A thin-walled cylindrical pressure vessel with a circular cross section is subjected to internal gas pressure p and simultaneously compressed by an axial load P = 12 k. The cylinder has inner radius r = 2.1 in and wall thickness t = 0.15 in.

  Determine the maximum allowable internal pressure based on an allowable shear stress of 6500 psi in the wall of the vessel



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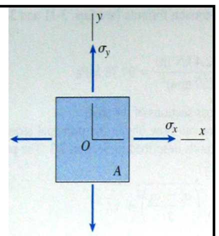
Solution



- 1) Select a *good* point: any point on surface
- 2) For each load, determine stress resultants:
  - Axial force: P: σ_{x1}
  - Twisting moment: none
  - Bending moment: none
  - Shear force: none
  - Internal pressure: p: σ_{x2} ,σ_{y2}

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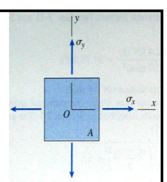
Solution (continued)



3) Calculate stresses

Axial Load:

$$\sigma_{x1} = -\frac{P}{A} = -\frac{P}{2\pi rt}$$

Internal Pressure:

$$\sigma_{x1} = \frac{pr}{2t}$$

$$\sigma_{y1} = \frac{pr}{t}$$

4) Combine individual stresses to find σ_x,σ_y, and τ_{xy}

$$\sigma_x = \sigma_{x1} + \sigma_{x2} = \frac{pr}{2t} - \frac{P}{2\pi rt}$$

$$\sigma_y = \frac{pr}{t}$$

$$\tau_{xy} = 0$$

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Solution (continued)

- Maximum Principal Stresses:
- No shear stresses – normal stresses are principal stresses

$$\sigma_1 = \sigma_y = \frac{pr}{t} = \frac{p(2.1in)}{0.15in} 14p$$

$$\sigma_2 = \sigma_x = \frac{pr}{2t} - \frac{P}{2\pi rt} = \frac{p(2.1in)}{2(0.15in)} - \frac{12k}{2\pi(2.1in)(0.15in)} = 7p - 6063psi$$

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Solution (continued)

- In plane shear stress:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = 3.5p + 3032psi$$

$$\tau_{\max} = 6500pi \Rightarrow p = 990psi$$

- Out-of-plane shear stress:

$$\tau_{\max} = \frac{\sigma_1}{2} = 7p \quad \Rightarrow p = 928psi$$

$$\tau_{\max} = \frac{\sigma_2}{2} = 3.5p - 3032psi \quad \Rightarrow p = 2720psi$$

Maximum pressure = 928 psi

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